Q3

a. $I = I_{inc}\cos^2\theta$ (Malus law) 0.5 pt

$I = (I_o/2)\cos^2\theta = I_o/8$ 0.5 pt

b. Let $T_{\|}$ and $T_{\perp}$ represent the energetic transmission coefficients of the polaroids parallel and perpendicular, respectively, to the transmission axis.

Paralel polaroids $I_{max} = (T_{\|}^2 + T_{\perp}^2)\, I_o/2$ 0.5 pt

Perpendicular polaroids $I_{min} = 2T_{\|}T_{\perp}\, I_o/2$ 0.5 pt

$$n = \frac{I_{max}}{I_{min}} = \frac{T_{||}^2 + T_{\perp}^2}{2T_{||}T_{\perp}}$$

The degree of polarization after a single polaroid:

$$P = \frac{T_{||} - T_{\perp}}{T_{||} + T_{\perp}} = \sqrt{\left(\frac{T_{||} - T_{\perp}}{T_{||} + T_{\perp}}\right)^2} = \sqrt{\frac{n-1}{n+1}}$$ 0.5 pt

The degree of polarization after two paralel polaroids:

$$P_{||} = \frac{T_{||}^2 - T_{\perp}^2}{T_{||}^2 + T_{\perp}^2} = \frac{T_{||} - T_{\perp}}{T_{||} + T_{\perp}}\frac{(T_{||} + T_{\perp})^2}{T_{||}^2 + T_{\perp}^2} = \frac{T_{||} - T_{\perp}}{T_{||} + T_{\perp}}\left(1 + \frac{2T_{||}T_{\perp}}{T_{||}^2 + T_{\perp}^2}\right) = \frac{\sqrt{n^2-1}}{n}$$ 0.5 pt

c. $E(P) = E_o/2$ 0.5 pt

d. $I(P) = E(P)E^*(P) = I_o/4$ 0.5 pt

e. $E(P) = \frac{1}{2}\frac{E_o}{\sqrt{2}}$ 0.5 pt

$I(P) = E(P)E^*(P) = I_o/8$ 0.5 pt

f. $E_{||}(P) = \frac{1}{2}\frac{E_o}{\sqrt{2}}$; $E_{\perp}(P) = \frac{1}{2}\frac{E_o}{\sqrt{2}}$ 0.5 pt

$I(P) = E_{\|}(P)E_{\|}^*(P) + E_{\perp}(P)E_{\perp}^*(P) = I_o/8 + I_o/8 = I_o/4$ 0.5 pt

g. $E_{||}(P) = \frac{E_o}{\sqrt{2}}$; $E_{\perp}(P) = \frac{1}{2}\frac{E_o}{\sqrt{2}}$ 0.5 pt

$I(P) = E_{\|}(P)E_{\|}^*(P) + E_{\perp}(P)E_{\perp}^*(P) = I_o/2 + I_o/8 = 5I_o/8$ 0.5 pt

h. $E_{||}(P) = \frac{1}{2}\alpha_{||}\frac{E_o}{\sqrt{2}} + \frac{1}{2}\alpha_{\perp}\frac{E_o}{\sqrt{2}}$; $E_{\perp}(P) = \frac{1}{2}\alpha_{||}\frac{E_o}{\sqrt{2}} + \frac{1}{2}\alpha_{\perp}\frac{E_o}{\sqrt{2}}$ 0.5 pt

$I(P) = E_{\|}(P)E_{\|}^*(P) + E_{\perp}(P)E_{\perp}^*(P) = (\alpha_{\|} + \alpha_{\perp})^2\, I_o/4 = I_o/4$ 0.5 pt

i. After the linearly polarized light goes through the half-wave plate, the light is still linearly polarised in a direction symmetrical to the optical axis of the half-wave plate. The contributions of the two areas will be in phase, but the angle between the two polarization directions at P will be 2θ: 0.5 pt

$$I(P) = \left(\frac{E_o}{2}\right)^2 + \left(\frac{E_o}{2}\right)^2 + 2\frac{E_o}{2}\frac{E_o}{2}\cos 2\theta = \frac{3E_o^2}{4} = \frac{3I_o}{4}$$ 0.5 pt

j. After the light goes through the two areas we have two beams that will be "out of phase". (the angle between the two components at P will be 180^o): 0.5 pt

$I(P) = 0$. 0.5 pt