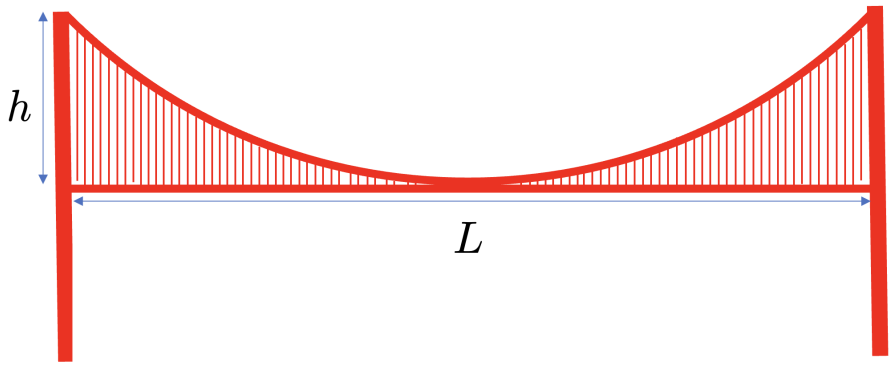
Problem 1. Building bridges

The goal of this problem is to analyze a variety of properties in suspension bridges, a type of bridge in which the deck is hung below a curved suspension cable using vertical suspenders.



To model the bridge, assume the following:

- There are two suspension cables, and the tension is distributed equally between the two. The weight of the suspension cables is much smaller than that of the deck, whose density per unit length is λ.
- In parts a) - c), assume that the weight of the vertical suspenders is negligible. Do not assume that in part d).
- The weight of the deck is uniformly supported by a large number of vertical suspenders for which the distance between consecutive suspenders is much smaller than the length of the bridge.
- The deck is perfectly horizontal.
- The suspension cable spans the length between two towers.
- The lowest point on the suspension cable is at the same vertical position as the deck.

Your task is to find the following:

a) Find an equation, $y(x)$, that describes the shape of the suspension cable. Your final result can depend on three constants that, in this part of the problem, can be left undetermined.

b) Assume that the two towers of the bridge have the same height h (above the suspension deck), and that the length of the deck in between the two towers is L. Assuming that $y = 0$ is the level of the deck and that $x = 0$ is the middle of the deck, find the shape of the suspension cable $y(x)$ solely in terms of L and h.

c) Find the maximum tension in the suspension cable in terms of the height of the bridge h, the length of the bridge L, the density per unit length λ, and the gravitational acceleration g. Where is the maximum tension achieved? Sktech this maximum tension for fixed L, in terms of the height h. What is the height of the towers h_{max} where the tension is maximized, and what is their height h_{min} when the tension is minimized?

d) The total weight of the vertical suspenders is initially much smaller than that of the deck. However, after a reconstruction project, the suspenders are reinforced, and their weight can no longer be neglected. Since the new suspenders are solid metal rods, you can assume that the shape of the suspension cable does not change. Assume that the total number of suspenders per unit length is n and that the mass per unit length of the suspenders is w, with $w \ll \lambda$. What is the new maximum tension in the suspension line? What height $h_{\text{min}}^{\text{new}}$ do the two towers need to be such that the maximum tension in the suspension cable is minimized?

Proposed by Luca Victor Iliesiu, *Stanford University*