Theoretical problem 1: Two paradoxes

A paradox is a statement that is apparently false or unveils its contradictory nature when analyzed in two different ways.

A. Where is the missing energy? *(4 points)*

A body (treated as a material point) with the mass m is kept at rest on a slope (Fig. 1) with a constant inclination on its whole length. When released, the body starts to slide towards the base of the slope and continues to move to the right on the horizontal plane. The connection region between the slope and the horizontal plane is smooth, such that the speed of the body does not change, only the direction of its velocity. The starting point is at a height h above the horizontal plane and the value of the gravitational acceleration is constant and known, g. Friction is neglected everywhere.

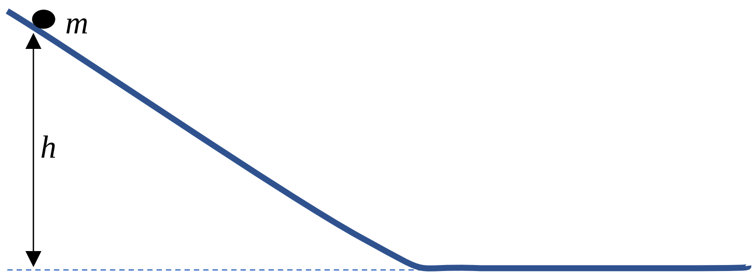

Fig. 1

A1	Derive the mathematical expression for the speed v_0 of the body on the horizontal plane.	0.30 p

Consider now the same process, but seen from a reference system moving to the right with the constant speed v_0 relative to the ground (the slope and the horizontal plane). In this reference system the final energy of the body is zero, while its initial energy is positive.

A2	Where did the energy “disappeared”? Give a detailed quantitative analysis of the missing energy.	3.70 p

B. Where is the missing angular momentum? *(6 points)*

B1. Momentum of electromagnetic waves

The energy transferred by an electromagnetic wave per unit of time per unit of surface is called Poynting vector and has the mathematical form

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B},$$

where μ_0 is the magnetic permeability of vacuum. The electric permittivity of vacuum, ε_0, is also known.

B1	Derive the volume density of the linear momentum of an electromagnetic wave, p_V. Express your result in vector form ($\vec{p}_V$).	0.90 p

B2. Feynman paradox

In Fig. 2 there are two long, coaxial cylindrical shells of length l. The inner one has the radius a and the electric charge $+Q$, uniformly distributed over its surface. The outer cylinder has the radius b $(b << l)$ and the electric charge $-Q$, uniformly distributed over its surface. The cylinders are made of the same material, having the mass per unit of area equal to σ. Coaxial with them there is a long solenoid with the radius R $(a < R < b)$, with n turns per unit length, and carrying an electric current i. The solenoid is fixed, but the cylindrical shells can freely and independently rotate around their common axis. Initially all the parts of this system are at rest.

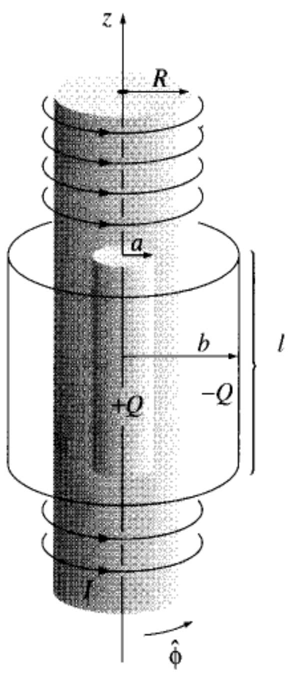

Fig. 2

B2.1. Angular velocities

B2.1	When the current in the solenoid is gradually reduced to zero, the cylinders begin to rotate. Derive the mathematical expressions of the final angular velocities (magnitude and orientation) of the cylinders.	2.60 p

Note: *The cylindrical shells are heavy enough to neglect any magnetic field due to their rotation!*

B2.2. Feynman paradox

B2.2	Since no external force acted on the system, its angular momentum should be conserved. From where did the angular momentum “appeared”? Give a detailed quantitative analysis of this paradox.	1.30 p

B2.3. Radial spoke

B2.3	Instead of decreasing the current through the solenoid, the cylinders are rigidly connected with a radial spoke with negligible mass (the practical method of doing this is not of interest here). The spoke is a weak conductor in order to neglect the displacement current. Determine the total angular momentum of the cylinders in this case, as well as their angular velocity.	1.20 p

proposed by

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