Problem III (10 points)

Rings and strips

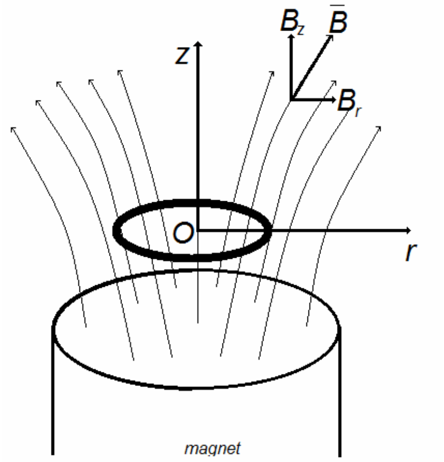
Task no. 1 - The rotating ring

A metal ring of radius R is made out of a wire of cross section aria s; the resistivity of the wire's material is given by ρ. The ring is place in a vertical plane. A small magnetic needle is placed in the center of the ring. The magnetic needle can rotate freely around the vertical axis which passes through the center of the ring. Due to mechanical inertia, the needle points in the direction of the *average horizontal magnetic* field at the center of the ring. When the ring is motionless in the reference frame of the Earth, the magnetic needle indicates the direction of the horizontal component of Earth's magnetic field. When the ring is rotating around its vertical axis with a constant angular velocity, ω, the magnetic needle deviates, in horizontal plane, from this direction by an angle α.

1.a. Find the expression of angle of deviation, α of the magnetic needle. Write your expression for α as a function of s, ω, ρ and of the magnetic permeability μ_0.

Task no. 2 - The superconducting ring

A thin ring of mass m, radius r_0 and inductance L is maintained in a horizontal plane above a cylindrical magnetic bar which is placed vertically (see figure). The vertical axis of symmetry of the cylindrical magnetic bar is aligned with the center of the ring. The magnetic field due to the cylindrical magnet is shown in the right figure and each of its components are given by: the radial component is given by $B_r = B_0 \cdot \beta \cdot r$, while the vertical component is given by $B_z = B_0 \cdot (1 - \alpha \cdot z)$. B_0, α, β are all positive constant with appropriate dimensions, while z and r denotes the vertical and, respectively, the radial coordinate of the system. Initially, there is no electric current passing through the ring and it is kept fixed above the magnet. It is then allowed to fall due to the gravitational pull of the Earth, given by the gravitational acceleration, g. During the fall the ring will still be in a horizontal plane and will have the same vertical axis.



Answer the following questions and write your results as a function of the variables specified above.

2.a. Derive the equations of motion for the ring in the reference frame specified in figure.

2.b. Find the expression of current $I = I(t)$ which flows through the ring at time t.

Task no. 3 - Conducting strips

For this task you are asked to determine a number of physical quantities for systems that contain one or two conducting strips which have an electric current flowing through them. These systems are placed in air $(\mu_{aer} \cong \mu_0)$.

Answer the following questions and write your results as a function of the variables specified below.

Through a conducting strip of width b and length D, $D>>b$ there is an electric current I flowing along its length. Assume that close to the strip the magnetic field lines are rectangles which surround the strip and are in planes orthogonal to it.

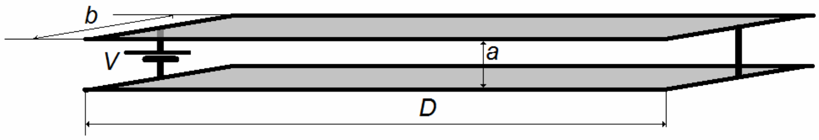
3.a. Find an expression which relates the induced magnetic field close to the strip to the intensity of electric current I.

We now add a second strip with the same length width D and width b and place it parallel, right above the first strip, at a distance a, $(a<<b<<D)$. The strips now both have a current I flowing along their length, but in opposite directions.

3.b. Find an expression which relates the induced magnetic field, $\vec{B}$, between the two strips to the electric current I.

3.c. Determine the inductance L of the two strip ensemble.

The two strips have negligible electric resistance and are incorporated in the circuit presented in the figure on the right. The capacitance of the whole ensemble is also negligible. The right side of the strips are connected by a wire with negligible resistivity while the sides on the left are connected to an ideal source, generating a voltage V. Neglect all effects due to the finite speed propagation of the electric field.



3.d. Find the time dependence of the electric current I from the circuit characterized only by the inductance L, found in 3.c. Assume the initial moment (t=0) is the moment at which the circuit is closed.

3.e. Denote the distance between a point on one of the strips and the right side of the strips by x. Determine the potential between this point and the point placed identically on the other strip as a function of x.

In an infinitesimal time interval dt the energy provided by the source is given by $dW=V\cdot I\cdot dt$. This amount of energy is partially stored in the magnetic field to the section x; the remaining energy „flows" from section to shorted ends of strips.

3.f. Find the expression of energy flux $dW(x)/dt$ as a function of the distance x between a point on the strip and the right end of the strip.

© Problems written by:

Prof. dr. Delia DAVIDESCU

Conf. univ. dr. Adrian DAFINEI

Answer sheet

Problem III (10 points)

Rings and strips

Task no. 1 - The rotating ring

1.a. The expression for the deviation angle of magnetic needle α during the rotation of ring.		2.60p

Task no. 2 - Superconducting ring

2.a. Expression for the equation of motion of the ring in the specified reference frame		4.00p
2.b. Formula for the current intensity $I = I(t)$ within the ring, as a function of time		0.40p

Task no. 3 – Conducting strips

3.a. Formula relating the magnetic field around the strip to the current intensity I within the strip.		0.50p
3.b. Formula relating the magnetic field $\vec{B}$ in between the two strips to the current intensity I within each strip (with the current traveling in opposite directions).		0.50p
3.c. Formula for the inductance L of the two strip system.		0.50p

3.d. Formula showing the time dependence of the current I within the circuit characterized by the previously determined inductance, L only.		0.50p
3.e. Expression for the voltage between two mirroring points located on the two strips as a function of the distance x .		0.50p
3.f. Formula for the energy flux $dW(x)/dt$ as a function of the distance x to the right end of the strips.		0.50p