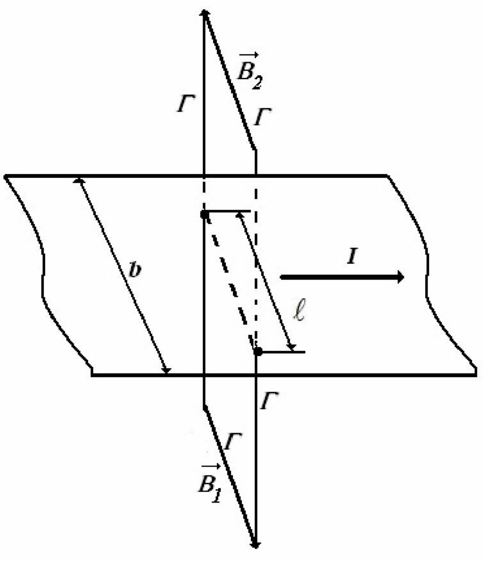
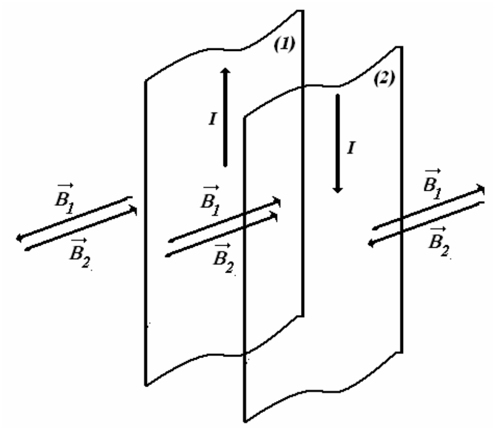
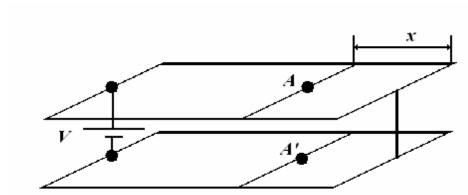
Marking scheme
Points are given for any correct solution

Problem III
Rings and strips

Nr. item	*Task no. 1 – Rotating ring*	Points
1.a.	For: $\vec{B}_{Earth} = B_h \cdot \hat{e}_x + B_v \cdot \hat{e}_z$ B_h - horizontal component of Earth's magnetic field B_v - vertical component of Earth's magnetic field 0.20p	2.60p
	formula of surface vector $\vec{S}$ for the ring $\vec{S} = \pi \cdot R^2 \cdot (\cos(\omega \cdot t)\cdot \hat{e}_x + \sin(\omega \cdot t)\cdot \hat{e}_y)$ 0.20p	
	expression of magnetic flux ϕ of Earth's magnetic field through ring's surface $\begin{cases} \phi = \vec{B}_{Earth} \bullet \vec{S} = (B_h \cdot \hat{e}_x + B_v \cdot \hat{e}_z)\bullet(\pi \cdot R^2 \cdot (\cos(\omega \cdot t)\cdot \hat{e}_x + \sin(\omega \cdot t)\cdot \hat{e}_y)) \\ \phi = B_h \cdot \pi \cdot R^2 \cdot \cos(\omega \cdot t) \end{cases}$ 0.40p	
	expression of electromotive force induced in the ring $\begin{cases} E = -\dfrac{d\phi}{dt} \\ E(t) = B_h \cdot \pi \cdot R^2 \cdot \omega \cdot \sin(\omega \cdot t) \end{cases}$ 0.20p	
	intensity of electrical current through the ring $i(t) = \dfrac{B_h \cdot s \cdot R \cdot \omega}{2 \cdot \rho} \cdot \sin(\omega \cdot t)$ 0.20p	
	modulus of induced magnetic field in center of ring $B_i(t) = \dfrac{\mu_0 \cdot B_h \cdot s \cdot R \cdot \omega}{4R \cdot \rho} \cdot \sin(\omega \cdot t)$ 0.20p	
	$\vec{B}_i = B_i \cdot (\hat{e}_x \cdot \cos(\omega \cdot t) + \hat{e}_y \cdot \sin(\omega \cdot t))$ $\vec{B}_i = \dfrac{\mu_0 \cdot B_h \cdot s \cdot \omega}{4 \cdot \rho} \cdot \left[\hat{e}_x \cdot \dfrac{1}{2} \cdot \sin(2\omega \cdot t) + \hat{e}_y \cdot \dfrac{1}{2} \cdot (1 - \cos(2\omega \cdot t))\right]$ 0.60p	
	average value per time of induced magnetic field $\langle \vec{B}_i \rangle = \dfrac{\mu_0 \cdot B_h \cdot s \cdot R \cdot \omega}{8R \cdot \rho} \cdot \hat{e}_y$ 0.20p	
	$tg\alpha = \dfrac{\lvert\langle \vec{B}_i \rangle\rvert}{B_h}$ 0.20p	
	$\alpha = arctg \dfrac{\mu_0 \cdot s \cdot \omega}{8 \cdot \rho}$ 0.20p	

Nr. item	*Task no. 2 – The superconducting ring*		Poins
2.a.	For: expression of magnetic flux through ring's surface $\Phi = B_z \cdot \pi \cdot r_0^2 + L \cdot I$	0.40p	4.00p
	$0 = R \cdot I = \frac{d\Phi}{dt}$ - voltage drop on superconducting ring is zero - magnetic flux inside the ring is constant	0.20p	
	$\Phi = B_0 \cdot (1 - \alpha \cdot z) \cdot \pi \cdot r_0^2 + L \cdot I = constant$ Initial conditions $\begin{cases} z(t=0) = 0 \\ I(t=0) = 0 \end{cases}$ $constant = B_0 \cdot \pi \cdot r_0^2$	0.40p	
	expression of the intensity of electric current through the ring $I = \frac{B_0}{L} \cdot \alpha \cdot \pi \cdot r_0^2 \cdot z$	0.40p	
	radial component of the force of interaction is zero - because of symmetry vertical component of the force of interaction $F_z = -\frac{2\pi^2 \cdot \alpha \cdot \beta \cdot B_0^2 \cdot r_0^4}{L} \cdot z$	0.60p	
	elastic constant $k = \frac{2\pi^2 \cdot \alpha \cdot \beta \cdot B_0^2 \cdot r_0^4}{L}$	0.20p	
	equations of motion for the ring $m \cdot \frac{d^2 z}{dt^2} + k \cdot z = -m \cdot g$	0.60p	
	general solution of the equations of motion for the ring $z(t) = A \cdot \cos(\omega \cdot t + \psi) - \frac{m \cdot g}{k}$	0.40p	
	initial conditions $\begin{cases} z(0) = 0 \\ \dot{z}(0) = v_z(0) = 0 \end{cases}$	0.20p	
	$\begin{cases} z(t) = \frac{m \cdot g \cdot L}{2\pi^2 \cdot \alpha \cdot \beta \cdot B_0^2 \cdot r_0^4} \cdot \cos\left[\sqrt{\frac{2\pi^2 \cdot \alpha \cdot \beta \cdot B_0^2 \cdot r_0^4}{L \cdot m}} \cdot t - 1\right] \\ z(t) = \frac{m \cdot g \cdot L}{2\pi^2 \cdot \alpha \cdot \beta \cdot B_0^2 \cdot r_0^4} \cdot \cos\left[\pi \cdot B_0 \cdot r_0^2 \cdot \sqrt{\frac{2 \cdot \alpha \cdot \beta}{L \cdot m}} \cdot t - 1\right] \end{cases}$ *Observations:* 1. Vertical coordinate is $z \le 0$. 2. Electric current passes all the time in the same direction through the ring and has a minimum value $(I = 0)$ in the upper point $(z = 0)$ of oscillation 3. The electromagnetic force is always upward with a minimum value $(F_m = 0)$ in the upper point of oscillation	0.60p	

2.b.	For: expression of time dependence of electric current $\begin{cases} I(t)=\dfrac{g\cdot m}{2\pi\cdot\beta\cdot B_0\cdot r_0^2}\cdot\left(\cos\sqrt{\dfrac{2\pi^2\cdot\alpha\cdot\beta\cdot B_0^2\cdot r_0^4}{L\cdot m}}\cdot t-1\right) \\ I(t)=\dfrac{g\cdot m}{2\pi\cdot\beta\cdot B_0\cdot r_0^2}\cdot\left(\cos\left[\pi\cdot B_0\cdot r_0^2\cdot\sqrt{\dfrac{2\cdot\alpha\cdot\beta}{L\cdot m}}\cdot t-1\right]\right) \end{cases}$ 0.40p	**0.40p**
Nr. item	*Task no. 3 – Conducting strips*	**Points**
3.a.	For:  Ampere law $\oint_\Gamma \vec{B}\cdot d\vec{r}=\mu_0\cdot\dfrac{I\cdot\ell}{b}$ $\begin{cases} B_1\cdot\ell+B_2\cdot\ell=\mu_0\cdot\dfrac{I\cdot\ell}{b} \\ B_1+B_2=\mu_0\cdot\dfrac{I}{b} \end{cases}$ 0.30p $B_1=B_2=\mu_0\cdot\dfrac{I}{2b}$ 0.20p	**0.50p**
3.b.	For :  $\begin{cases} B=B_1+B_2 \\ B=\mu_0\cdot\dfrac{I}{b} \end{cases}$ 0.50p	**0.50p**

3.c.	For: $\begin{cases} U = -\dfrac{d\varphi}{dt} \\ U = -L \cdot \dfrac{dI}{dt} \end{cases}$ or $\varphi = L \cdot I$	0.30p	**0.50p**
	$L = \dfrac{\mu_0 \cdot D \cdot a}{b}$	0.20p	
3.d.	For: the second II Kirchhoff law $V = L \cdot \dfrac{dI}{dt}$ *(R=0, C=0)*	0.30p	**0.50p**
	$I(t) = \dfrac{V \cdot b}{\mu_0 \cdot D \cdot a} \cdot t$	0.20p	
3.e.	For:  Voltage difference $U_{AA'}$ between two mirroring points of the two bands is due to self-induced voltage corresponding to the portion starting at x to the end of the assembly strip $U_{AA'} = L(x) \cdot \dfrac{dI}{dt}$ $L(x)$ - the inductance linked with of the magnetic flux through ending portion (of length x) of the strip assembly $\begin{cases} L(x) = \dfrac{\Phi(x)}{I} \\ L(x) = \dfrac{B \cdot x \cdot a}{I} \end{cases}$ $L(x) = \dfrac{\mu_0 \cdot x \cdot a}{b}$	0.30p	**0.50p**
	$U_{AA'} = V \cdot \dfrac{x}{D}$	0.20p	
3.f.	For: $\begin{cases} \dfrac{dW}{dt} = I \cdot U_{AA'} \\ \dfrac{dW}{dt} = \dfrac{V^2 \cdot b \cdot x}{\mu_0 \cdot D^2 \cdot a} \cdot t \end{cases}$	0.50p	**0.50p**
TOTAL Problem III			**10p**

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