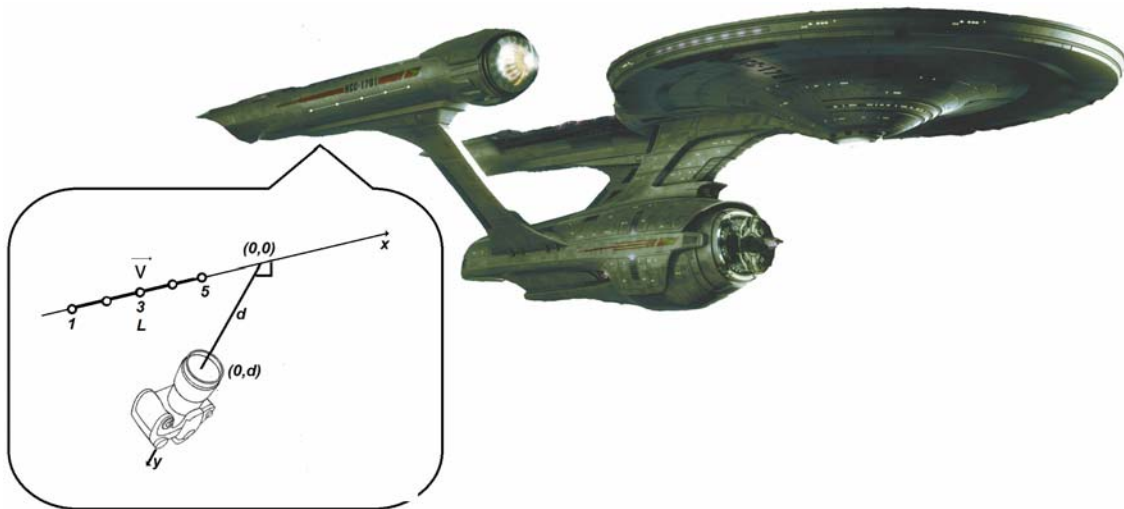


Theoretical Problem No. 3 - Star Trek Watching (10 points)

Description of the situation



Star Ship Enterprise has on each of the two warp engines a succession of five collinear, equidistant, numbered, beacons as shown in the figure (for the propeller on right side). When traveling, SS Enterprise moves with constant speed \bar{v} so that, for the questions that follow, beacons located on a propeller are always on the same line that is Ox axis of the coordinate system. In the situation room of another spacecraft which is at rest and which oversees the SS Enterprise, are analyzed the images of line of beacons. The images are taken by a camera with the aperture located at the point $(0,d)$ - in the orthogonal coordinate system shown in the inset in figure above. An image of line of beacons is produced by light rays that arrive simultaneously at camera aperture that opens for a short time. On a picture of SS Enterprise at rest, the length of line of beacons is L . In the problem, the beacons can be considered as points.

The speed of light is $c = 3,00 \times 10^8 \text{ m} \cdot \text{s}^{-1}$.

Employ the quantities $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$, if they help to simplify your result.

Task No. 1 - The link between image and the actual position

The actual position is the position in the frame in which the camera is at rest.

Next, the camera remains stationary in the point $(0,d)$ and the SS Enterprise is moving at velocity \bar{v} on Ox direction. In an image provided by the camera described above is observed that one of the bright beacons is placed in the position x_i .

- 1.a.** Determine the expression for the actual position of the bright beacon when the image is formed in the camera.
- 1.b.** Find also the corresponding inverse relation, that is the expression of x_i in terms of x, d, L, v and c .

Task No. 2 - Apparent length of the beacons line

The camera takes a picture of the instant when the actual position of the center of beacons line is at some x_0 point.

- 2.a.** Determine the expression for the apparent length of the beacons line on this picture.
- 2.b.** Indicate – using a mathematical expression – how the apparent length of beacons line changes with time, when SS Enterprise moves along the Ox axe. The space ship comes from far away, passes through the origin of the system and then receding far away.

Task No. 3 - Symmetrical image

One of the pictures taken by the camera shows both ends of beacons line at the same distance from the aperture of camera.

- 3.a.** Determine the expression for the apparent length of beacons line on this picture.
- 3.b.** Determine the expression for the actual position of the middle beacons at the time when this picture is taken.
- 3.c.** Determine where will be situated on the picture the image of the middle beacons.

Task No. 4 - Pictures of SS Enterprise, being far away and approaching or being far away and receding

Camera takes a picture when SS Enterprise is approaching from far away and another image when the ship (and the beacons line) is far away and receding. On one of these images the apparent length of the line of beacons is 160 m and the other is 600 m .

- 4.a.** Decide which of the following statements is correct.
- i.** Apparent length is 200 m on the image of „approaching” ship and 600 m on the image of „receding” ship.
- ii.** Apparent length is 600 m on the image of „approaching” ship and 200 m on the image of „receding” ship.

Write in the appropriate box in the answer sheet the letter corresponding to the answer you think is correct. Briefly justify your choice.

- 4.b.** Determine the speed v of the ship.
- 4.c.** Calculate the value of length of the beacons line at rest.
- 4.d.** Calculate the value of length of the beacons line on the symmetric picture, described at task no. 3.a.

© The Problem is proposed by:

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ANSWER SHEET

Theoretical Problem No. 3 - Star Trek Watching (10 points)

Task No. 1 - The link between image and the actual position

1.a. The expression for the actual position of the bright beacon when the image is formed in the camera

1,00p

1.b. Corresponding inverse relation, that is the expression of x_i in terms of x, d, L, v and c

1,00p

Task No. 2 - Apparent length of the beacons line

2.a. The expression for the apparent length of the beacons line on this picture

1,00p

2.b. Indicate – using a mathematical expression – how the apparent length of beacons line changes with time, when SS Enterprise moves along the Ox axe.

1,00p

Task No. 3 - Symmetrical image

3.a. The expression for the apparent length of beacons line on this picture

1,00p

3.b. The expression for the actual position of the middle beacons at the time when this picture is taken

1,00p

3.c. Determine where will be situated on the picture the image of the middle beacons

1,00p

Task No. 4 - Pictures of SS Enterprise, being far away and approaching or being far away and receding

4.a. Decide which of the following statements is correct.

- i. Apparent length is 200m on the image of „approaching” ship and 600m on the image of „receding” ship.
- ii. Apparent length is 600m on the image of „approaching” ship and 200m on the image of „receding” ship.

Write in the appropriate box in the answer sheet the letter corresponding to the answer you think is correct. Briefly justify your choice.

1,00p

4.b. The speed v of the ship

1,00p

4.c. The value of length of the beacons line at rest

0,50p

4.d. The value of length of the beacons line on the symmetric picture

0,50p