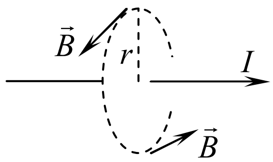
PROBLEM No. 1

a. 1p

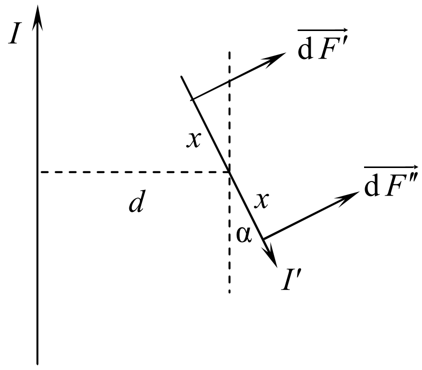
From the symmetry of the situation we take the magnetomotive force along a circular path of radius r, centered on the wire.



$$\oint \vec{B}\,\mathrm{d}\vec{l} = 2\pi r B = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

b. 1.5p

Consider two elements dx of the rod placed symmetrically at distances x from its center. The corresponding forces acting on them are:



$$\mathrm{d}F' = \frac{\mu_0 I I' \,\mathrm{d}x}{2\pi\left(d - x\sin\alpha\right)}$$

$$\mathrm{d}F'' = \frac{\mu_0 I I' \,\mathrm{d}x}{2\pi\left(d + x\sin\alpha\right)}$$

The sum of their torques is:

$$\mathrm{d}M = \left(\mathrm{d}F'' - \mathrm{d}F'\right)x = -\frac{2\mu_0 I I' x^2 \sin\alpha \,\mathrm{d}x}{2\pi\left(d^2 - x^2\sin^2\alpha\right)}$$

For very small angles, the total torque is:

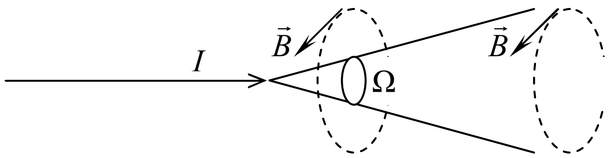
$$M = \int_0^{L/2} -\frac{\mu_0 I I' \alpha x^2 dx}{\pi d^2} = -\frac{\mu_0 I I' \alpha L^3}{24\pi d^2} = \frac{mL^2}{12}\ddot{\alpha} \Rightarrow T_{\text{slant}} = 2\pi d\sqrt{\frac{2\pi m}{\mu_0 I I' L}}$$

c. 1p

Taking path integrals along circular field lines exactly like at the first point, we get:

$$B_{\text{IN}} = 0$$

$$B_{\text{OUT}} = \frac{\mu_0 I}{2\pi r}$$

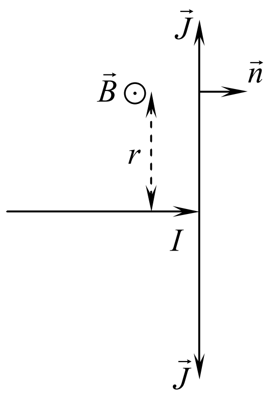


d. 0.5p

The above argument keeps holding, and the results are:

$$B_{\text{WIRE SIDE}} = \frac{\mu_0 I}{2\pi r}$$

$$B_{\text{OTHER SIDE}} = 0$$



e. 1p

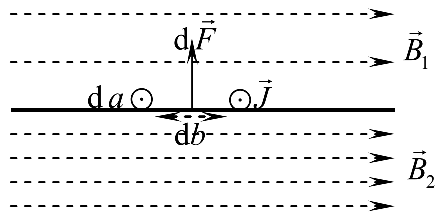
$$J(r) = \frac{I}{2\pi r}$$

$$\Delta B_{\parallel} = B_{\parallel\text{OTHER SIDE}} - B_{\parallel\text{WIRE SIDE}} = 0 - \left(-\frac{\mu_0 I}{2\pi r}\right) = \mu_0 J$$

f. 1.5p

Consider a small region of the plane having dimensions da along J and db across J.

$$J = \frac{B_2 - B_1}{\mu_0}$$



Let B_0 be the external magnetic field and B' the field generated by the conducting plane.

$$\left.\begin{aligned} B_1 &= B_0 - B' \\ B_2 &= B_0 + B' \end{aligned}\right\} \Rightarrow B_0 = \frac{B_1 + B_2}{2}$$

$$\mathrm{d}F = \mathrm{d}I \cdot \mathrm{d}a \cdot B_0 = J \cdot \mathrm{d}b \cdot \mathrm{d}a \frac{B_1 + B_2}{2} = \frac{B_2 - B_1}{\mu_0} \mathrm{d}S \frac{B_1 + B_2}{2} \Rightarrow p = \frac{\mathrm{d}F}{\mathrm{d}S} = \frac{B_2^2 - B_1^2}{2\mu_0}$$

g. 0.5p

Just as before,

$$B_{\mathrm{IN}} = 0$$

$$B_{\mathrm{OUT}} = \frac{\mu_0 I}{2\pi r}$$

h. 1p

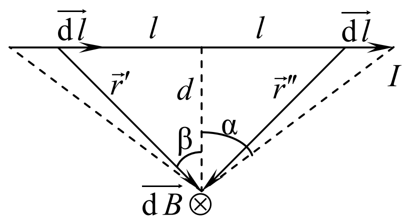
This time the path integrals go the other way around.

$$B_{\mathrm{IN}} = \frac{\mu_0 I}{2\pi r}$$

$$B_{\mathrm{OUT}} = 0$$

i. 1p

Consider two elements dl of the wire, placed symmetrically at a distance l from the center of the wire. Their contributions to the magnetic field in the mediator plane are equal:



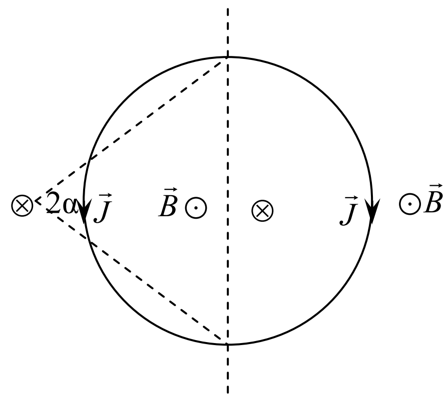
$$\left|\mathrm{d}\vec{B}\right| = \frac{\mu_0 I \,\mathrm{d}l \sin(90° - \beta)}{4\pi r^2} = \frac{\mu_0 I \,\mathrm{d}l}{4\pi d^2} \cos^3 \beta$$

$$l = d \tan\beta \Rightarrow dl = \frac{d}{\cos^2\beta} d\beta \Rightarrow \left|\mathrm{d}\vec{B}\right| = \frac{\mu_0 I}{4\pi d} \cos\beta \,\mathrm{d}\beta$$

$$B = 2\int_0^\alpha \frac{\mu_0 I}{4\pi d} \cos\beta d\beta = \frac{\mu_0 I}{2\pi d} \sin\beta \Big|_0^\alpha = \frac{\mu_0 I}{\pi L} \frac{\sin^2\alpha}{\cos\alpha}$$

j. 1p

From a point in the equatorial plane, the axis of the poles of the sphere is seen under an angle 2α, with tan α = R/r.



Outside, the sphere behaves similarly to an electric current flowing directly from one pole to the other through a wire connecting the poles directly:

$$B_{\mathrm{OUT}}(r) = \frac{\mu_0 I}{2\pi} \frac{R}{r} \frac{1}{\sqrt{r^2 + R^2}}$$

Inside, the sphere behaves similarly to two semi-infinite straight conductors connecting the two poles of the sphere and carrying the current I in the opposite direction:

$$B_{\mathrm{IN}}(r) = \frac{\mu_0 I}{2\pi r}\left(1 - \frac{R}{\sqrt{r^2 + R^2}}\right)$$

PROBLEM No. 2

a. 1p

Consider an element dx of the rod, placed at distance x from the center of the rod. Its mass is dm = mdx/L each. Let 2l be the length of the rod at some moment, and let y be the corresponding length of the region x.

Since the object is homogenous at all times,

$$\frac{y}{x}=\frac{l}{L/2}$$

Let v be the velocity of the two ends of the rod at some moment.

$$v=\frac{\mathrm{d}l}{\mathrm{d}t}=\frac{\mathrm{d}(2l-L)}{2\,\mathrm{d}t}=\frac{\mathrm{d}\left(\frac{2l-L}{L}\right)}{\frac{2\,\mathrm{d}t}{L}}=\frac{L\,\mathrm{d}\varepsilon}{2\,\mathrm{d}t}=\frac{L}{2}\dot{\varepsilon}$$

The velocity of the element considered is

$$v(x)=\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\mathrm{d}\left(\frac{lx}{L/2}\right)}{\mathrm{d}t}=\frac{x}{L/2}\frac{dl}{dt}=\frac{xv}{L/2}=\frac{2x}{L}\frac{L\dot{\varepsilon}}{2}=x\dot{\varepsilon}$$

The kinetic energy of the rod is

$$E_{\text{kin}}=2\int_0^{L/2}\frac{dmv^2(x)}{2}=\int_0^{L/2}x^2\dot{\varepsilon}^2\frac{mdx}{L}=\frac{m\dot{\varepsilon}^2}{L}\frac{x^3}{3}\bigg|_0^{L/2}=\frac{mL^2\dot{\varepsilon}^2}{24}$$

b. 0.5p

Let S be the cross section of the rod, and V its volume. The elementary work done by the tensile force σS equals the increase in elastic potential energy.

$$\mathrm{d}E_{\text{pot}}=\mathrm{d}W=F\,\mathrm{d}(2l)=\sigma S\,\mathrm{d}(2l-L)=E\varepsilon\frac{V}{L}\mathrm{d}(2l-L)=\frac{mE}{\rho}\varepsilon\,\mathrm{d}\varepsilon=\mathrm{d}\left(\frac{mE}{2\rho}\varepsilon^2\right)\Rightarrow$$

$$E_{\text{pot}}=\frac{mE\varepsilon^2}{2\rho}$$

c. 0.5p

$$E_{\text{mech}}=E_{\text{kin}}+E_{\text{pot}}=\frac{mL^2\dot{\varepsilon}^2}{24}+\frac{mE\varepsilon^2}{2\rho}=\text{constant}\Rightarrow\dot{E}_{\text{mech}}=\frac{mL^2\dot{\varepsilon}\ddot{\varepsilon}}{12}+\frac{mE\varepsilon\dot{\varepsilon}}{\rho}=0\Rightarrow$$

$$\frac{mL^2\ddot{\varepsilon}}{12}+V\sigma=0$$

Dividing by L we get:

$$m\left(\frac{L\varepsilon}{12}\right)^{\cdot\cdot}=-\sigma S\Rightarrow\ddot{x}_{\text{equivalent}}=\left(\frac{L\varepsilon}{12}\right)^{\cdot\cdot}$$

d. 0.5p

Dividing also by m we get:

$$\frac{L^2}{12}\ddot{\varepsilon}+\frac{E}{\rho}\varepsilon=0\Rightarrow\omega^2=\frac{12E}{\rho L^2}\Rightarrow T_{\text{long}}=\pi L\sqrt{\frac{\rho}{3E}}$$

e. 0.5p

Consider very thin spherical layers of radius x and thickness dx. Their masses are:

$$\mathrm{d}m = m\frac{4\pi x^2 \mathrm{d}x}{\frac{4\pi R^3}{3}} = \frac{3m}{R^3}x^2\,\mathrm{d}\,x$$

Let r be the radius of the sphere and v the velocity of its surface at some moment. The argument goes similarly as in section A.

$$v = R\dot{\varepsilon} \Rightarrow v(x) = x\dot{\varepsilon} \Rightarrow \mathrm{d}\,E_{\mathrm{kin}} = \frac{\mathrm{d}\,mv^2(x)}{2} = \frac{3mx^2\,\mathrm{d}\,x}{R^3}\frac{x^2\dot{\varepsilon}^2}{2} \Rightarrow E_{\mathrm{kin}} = \frac{3mR^2\dot{\varepsilon}^2}{10}$$

$$\mathrm{d}\,E_{\mathrm{pot}} = \mathrm{d}\,W = \sigma S\,\mathrm{d}\,r = \varepsilon E 4\pi R^2 R\,\mathrm{d}\,\varepsilon = \frac{3EV\,\mathrm{d}\left(\varepsilon^2\right)}{2} = \mathrm{d}\left(\frac{3mE}{2\rho}\varepsilon^2\right)$$

$$E_{\mathrm{mech}} = \frac{3m}{2}\left(\frac{R^2\dot{\varepsilon}^2}{5} + \frac{E\varepsilon^2}{\rho}\right) = \text{constant}$$

f. 0.5p

$$\dot{E} = 0 \Rightarrow \frac{R^2\dot{\varepsilon}\ddot{\varepsilon}}{5} + \frac{E\varepsilon\dot{\varepsilon}}{\rho} = 0 \Rightarrow \omega^2 = \frac{5E}{\rho R^2} \Rightarrow T_{\mathrm{radial}} = 2\pi R\sqrt{\frac{\rho}{5E}}$$

g. 0.5p

$$\begin{cases} \varepsilon_x = \dfrac{\sigma_x}{E} - \mu\dfrac{\sigma_y}{E} \\ \varepsilon_y = \dfrac{\sigma_y}{E} - \mu\dfrac{\sigma_x}{E} \end{cases} \Rightarrow \begin{cases} \sigma_x = \dfrac{E\left(\varepsilon_x + \mu\varepsilon_y\right)}{1-\mu^2} \\ \sigma_y = \dfrac{E\left(\varepsilon_y + \mu\varepsilon_x\right)}{1-\mu^2} \end{cases}$$

h. 0.5p

$$\begin{cases} m\ddot{x}_{\mathrm{equivalent}} = -\sigma_x\dfrac{V}{L} \\ m\ddot{y}_{\mathrm{equivalent}} = -\sigma_y\dfrac{V}{l} \end{cases} \Rightarrow \begin{cases} \dfrac{mL\ddot{\varepsilon}_x}{12} + \dfrac{E\left(\varepsilon_x + \mu\varepsilon_y\right)}{1-\mu^2}\dfrac{V}{L} = 0 \\ \dfrac{ml\ddot{\varepsilon}_y}{12} + \dfrac{E\left(\varepsilon_y + \mu\varepsilon_x\right)}{1-\mu^2}\dfrac{V}{l} = 0 \end{cases}$$

i. 1.5p

By replacing the sought solutions into the system of equations we get

$$\begin{cases} -\dfrac{\omega^2 AL^2}{12} + \dfrac{E\left(A + \mu B\right)}{\rho\left(1-\mu^2\right)} = 0 \\ -\dfrac{\omega^2 Bl^2}{12} + \dfrac{E\left(B + \mu A\right)}{\rho\left(1-\mu^2\right)} = 0 \end{cases}$$

By dividing the two equations term by term we get a simpler one:

$$\frac{AL^2}{Bl^2} = \frac{A + \mu B}{B + \mu A}$$

Let us denote the ratio of the two amplitudes by r.

$$r\frac{L^2}{l^2}=\frac{r+\mu}{1+r\mu}\Rightarrow \mu L^2r^2+\left(L^2-l^2\right)r-\mu l^2=0\Rightarrow$$

$$r_{1;2}=\frac{-\left(L^2-l^2\right)\pm\sqrt{\left(L^2-l^2\right)^2+4\mu^2L^2l^2}}{2\mu L^2}$$

Returning r in the second equation we get:

$$\omega^2=\frac{12E}{\rho l^2\left(1-\mu^2\right)}\left[1+\mu\frac{-\left(L^2-l^2\right)\pm\sqrt{\left(L^2-l^2\right)^2+4\mu^2L^2l^2}}{2\mu L^2}\right]\Rightarrow$$

$$\omega_{1;2}=\sqrt{\frac{6E\left[L^2+l^2\pm\mu\sqrt{\left(L^2-l^2\right)^2+\left(2\mu Ll\right)^2}\right]}{\rho L^2l^2\left(1-\mu^2\right)}}$$

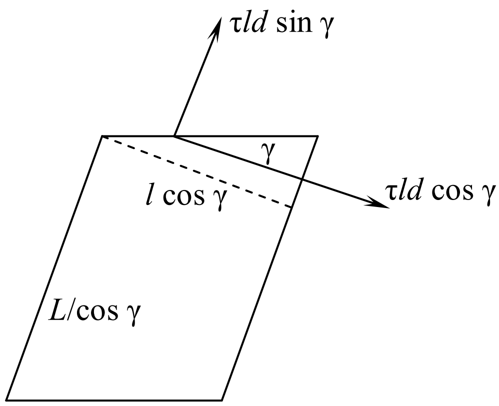
j. 0.5p

$$L=l\Rightarrow\omega_{1;2}=\sqrt{\frac{6E\left(2L^2\pm2\mu^2L^2\right)}{\rho L^4\left(1-\mu^2\right)}}=\sqrt{\frac{12E\left(1\pm\mu^2\right)}{\rho L^2\left(1-\mu^2\right)}}$$

$$\Delta\omega=\sqrt{\frac{12E}{\rho L^2}}\left(\sqrt{\frac{1+\mu^2}{1-\mu^2}}-1\right)\approx\mu^2\sqrt{\frac{12E}{\rho L^2}}\Rightarrow T_{\text{beats}}=\frac{T_{\text{long}}}{\mu^2}$$

k. 1.5p

Let d be the thickness of the plate. The shear force τld can be decomposed into a stretching component along L (x-axis) and a shrinking component along l (y-axis).



$$\sigma_x=\frac{\tau ld\sin\gamma}{ld}\;;\;\sigma_y=\frac{\tau ld\cos\gamma}{\left(L/\cos\gamma\right)d}\Rightarrow$$

$$\varepsilon_x=\frac{\tau\sin\gamma}{E}-\mu\left(-\frac{\tau l\cos^2\gamma}{LE}\right)$$

$$\varepsilon_y=-\frac{\tau l\cos^2\gamma}{LE}-\mu\frac{\tau\sin\gamma}{E}$$

But

$$\varepsilon_x=\frac{\frac{L}{\cos\gamma}-L}{L}=\frac{1-\cos\gamma}{\cos\gamma}\;;\;\varepsilon_y=\frac{l\cos\gamma-l}{l}=-\left(1-\cos\gamma\right)\Rightarrow$$

$$\begin{cases}\dfrac{E}{\tau}\dfrac{1-\cos\gamma}{\cos\gamma}=\sin\gamma+\mu\dfrac{l}{L}\cos^2\gamma\\[2mm]\dfrac{E}{\tau}\left(1-\cos\gamma\right)=\dfrac{l}{L}\cos^2\gamma+\mu\sin\gamma\end{cases}$$

Multiplying the second equation by μ and subtracting it from the first one we get:

$$\frac{E}{\tau}\left(1-\cos\gamma\right)\left(\frac{1}{\cos\gamma}-\mu\right)=\sin\gamma\left(1-\mu^2\right)\Rightarrow\frac{E\gamma^2}{2\tau}\left(1-\mu\right)\approx\gamma\left(1-\mu^2\right)\Rightarrow\gamma=\frac{2\tau\left(1+\mu\right)}{E}$$

$$G = \frac{E}{2(1+\mu)}$$

l. 0.5p

The quantities involved in the shear deformation are absolutely analogous to those describing the longitudinal deformation.

$$T_{\text{slant}} = \pi L\sqrt{\frac{\rho}{3G}} = T_{\text{long}}\sqrt{2(1+\mu)}$$

m. 0.5p

Consider very thin cylindrical layers of radius x and thickness dx. When the cylinder is twisting, each one of them is subject to a very small shear.

$$T_{\text{twist}} = \pi L\sqrt{\frac{\rho}{3G}}$$

n. 1p

Let α be a very small angle with witch one cap of the cylinder rotates with respect to the other. Then the slanting angle of a cylindrical layer is:

$$x\alpha = L\gamma \Rightarrow \gamma = \frac{x}{L}\alpha$$

The corresponding shear stress is

$$\tau = G\frac{x}{L}\alpha$$

The elementary shear force acting on the cap is

$$\mathrm{d}F = \tau\,\mathrm{d}S = G\frac{x}{L}\alpha 2\pi x\,\mathrm{d}x$$

The corresponding elementary torque is

$$\mathrm{d}M = \mathrm{d}F \cdot x = \frac{2\pi G\alpha x^3\mathrm{d}x}{L}$$

$$M = \frac{2\pi G\alpha}{L}\int_0^R x^3\,\mathrm{d}x = \frac{2\pi GR^4\alpha}{4L} \Rightarrow C = \frac{\pi GR^4}{2L} = \frac{\pi ER^4}{4L(1+\mu)}$$

PROBLEM No. 3

a. 0.5p

Deriving the Lorentz transformations two-fold, we get

$$a_x = a'_x \left(\frac{\sqrt{1-\frac{u^2}{c^2}}}{1+\frac{u}{c^2}v'_x} \right)^3$$

In our case $u = v_x$ and $v_x' = 0$.

$$\frac{\mathrm{d}v_x}{\mathrm{d}t} = a'\left(1-\frac{v_x^2}{c^2}\right)^{\frac{3}{2}}$$

$$F_x = \frac{ma_x}{1-\frac{v_x^2}{c^2}} = m_0 a' = \text{constant}$$

b. 0.5p

$$v_x = c\sin\alpha \Rightarrow \frac{\mathrm{d}(c\sin\alpha)}{\left(1-\sin^2\alpha\right)^{\frac{3}{2}}} = a'\mathrm{d}t \Rightarrow c\tan\alpha = a't + C$$

At $t = 0$, $v_x = 0$, so $\alpha = 0$ and $C = 0$.

$$\frac{\frac{v_x}{c}}{\sqrt{1-\frac{v_x^2}{c^2}}} = \frac{a't}{c} \Rightarrow v = c\frac{\frac{a't}{c}}{\sqrt{1+\left(\frac{a't}{c}\right)^2}}$$

c. 0.5p

$$\mathrm{d}t' = \mathrm{d}t\sqrt{1-\frac{v^2}{c^2}} = \frac{\mathrm{d}t}{\sqrt{1+\left(\frac{a't}{c}\right)^2}};\ \frac{a't}{c} = \sinh\tau \Rightarrow \mathrm{d}t' = \frac{c}{a'}\mathrm{d}\tau \Rightarrow t' = \frac{c}{a'}\tau + C$$

Again $C = 0$, so

$$t' = \frac{c}{a'}\operatorname{arcsinh}\left(\frac{a't}{c}\right)$$

d. 1p

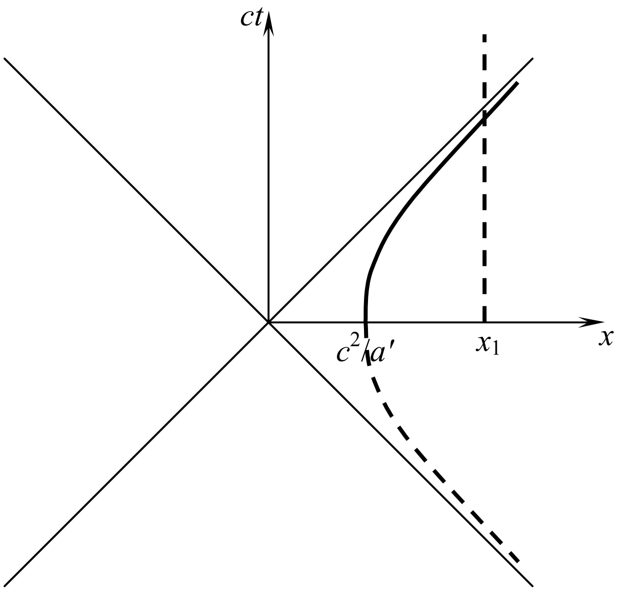
$$\left.\begin{aligned} -c^2(\mathrm{d}t')^2 &= -c^2(\mathrm{d}t)^2 + (\mathrm{d}x)^2 \\ \frac{a't}{c} = \sinh\tau &\Rightarrow \mathrm{d}t = \frac{c}{a'}\cosh\tau\,\mathrm{d}\tau \end{aligned}\right\} \Rightarrow (\mathrm{d}x)^2 = \frac{c^4}{a'^2}\left(\cosh^2\tau - 1\right)(\mathrm{d}\tau)^2 \Rightarrow dx = \frac{c^2}{a'}\sinh\tau\,\mathrm{d}\tau \Rightarrow$$

$$x = \frac{c^2}{a'}\cosh\tau + C$$

At $t = t' = 0$, $x_0 = c^2/a'$, so again $C = 0$.

e. 1p

$$ct=\frac{c^2}{a'}\sinh\tau \Rightarrow x^2-(ct)^2=\left(\frac{c^2}{a'}\right)^2 \Rightarrow \frac{x^2}{\left(\frac{c^2}{a'}\right)^2}-\frac{(ct)^2}{\left(\frac{c^2}{a'}\right)^2}=1$$



f. 0.5p

$$\rho_0=\frac{c^2}{a'}\Rightarrow\begin{cases}x=\rho_0\cosh\tau\\ ct=\rho_0\sinh\tau\end{cases}$$

g. 0.5p

$$\begin{cases}x=\rho\cosh\tau\\ ct=\rho\sinh\tau\end{cases};\quad\begin{cases}\rho=\sqrt{x^2-(ct)^2}\\ \tau=\operatorname{arctanh}\left(\frac{ct}{x}\right)\end{cases}$$

These equations require that $x > 0$ and $\rho > 0$, so using these new parameters one can cover only the quadrant of spacetime characterized by $x > |ct|$.

h. 1p

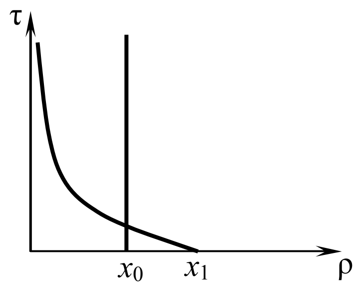
$$\left.\begin{aligned}\mathrm{d}x&=\mathrm{d}\rho\cosh\tau+\rho\sinh\tau\,\mathrm{d}\tau\\ \mathrm{d}(ct)&=c\,\mathrm{d}t=\mathrm{d}\rho\sinh\tau+\rho\cosh\tau\,\mathrm{d}\tau\end{aligned}\right\}\Rightarrow$$

$$ds^2=-c^2(\mathrm{d}t)^2+(\mathrm{d}x)^2=(\mathrm{d}\rho)^2-\rho^2(\mathrm{d}\tau)^2=-c^2\frac{\rho^2}{c^2}(\mathrm{d}\tau)^2+(\mathrm{d}\rho)^2\ ;\ f=\frac{\rho^2}{c^2}\ ;\ g=1$$

i. 0.5p

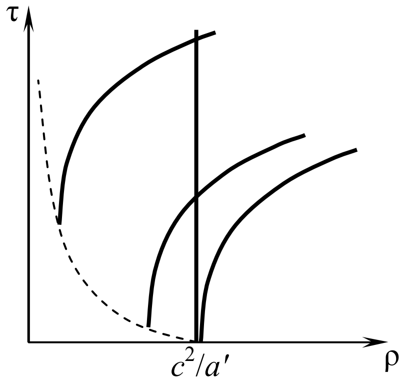
$$\rho=\frac{x_1}{\cosh\tau}\Leftrightarrow\tau=\operatorname{arccosh}\left(\frac{x_1}{\rho}\right)$$

$$\Delta\rho=\frac{c^2}{a'}$$



j. 0.5p

The observer will receive only those signals emitted before the beacon exits the quadrant of spacetime described by the Rindler metric.



$$ct \le x = x_0 \Rightarrow t_{\lim} = \frac{x_0}{c} \Rightarrow N = \left[\frac{t_{\lim}}{T_0}\right] + 1 = \left[\frac{x_0}{cT_0}\right] + 1$$

In the case of the light,

$$ds^2 = 0 \Rightarrow (\mathrm{d}\rho)^2 = \rho^2 (\mathrm{d}\tau)^2 \Rightarrow \frac{\mathrm{d}\rho}{\rho} = \mathrm{d}\tau$$

At $\tau = 0$, $\rho_0 = x_0$, so

$$\ln\frac{\rho}{\rho_0} = \tau \Rightarrow \rho = x_0\, \mathrm{e}^{\tau}$$

k. 1.5p

Let ρ_e and τ_e be the spacetime coordinates for the emission of a pulse.

$$\rho_e = \sqrt{x_0^2 - c^2t^2}\ ;\ \tau_e = \operatorname{arctanh}\left(\frac{ct}{x_0}\right)$$

$$\tanh\tau_e = \frac{\mathrm{e}^{\tau_e} - \mathrm{e}^{-\tau_e}}{\mathrm{e}^{\tau_e} + \mathrm{e}^{-\tau_e}} = \frac{ct}{x_0} \Rightarrow \mathrm{e}^{2\tau_e} - 1 = \frac{ct}{x_0}\left(\mathrm{e}^{2\tau_e} + 1\right) \Rightarrow \mathrm{e}^{2\tau_e} = \frac{1 + \frac{ct}{x_0}}{1 - \frac{ct}{x_0}} \Rightarrow \mathrm{e}^{\tau_e} = \sqrt{\frac{x_0 + ct}{x_0 - ct}}$$

$$\rho = \frac{\rho_e}{\mathrm{e}^{\tau_e}}\mathrm{e}^{\tau} = (x_0 - ct)\mathrm{e}^{\tau} = \rho_0 \Rightarrow \mathrm{e}^{\tau} = \frac{x_0}{x_0 - ct} \Rightarrow \tau = \ln\left(\frac{x_0}{x_0 - ct}\right)$$

Let t^* be the moment the observer receives the last signal.

$$v(t^*) = c\frac{\frac{a't^*}{c}}{\sqrt{1 + \left(\frac{a't^*}{c}\right)^2}}$$

The frequency received is

$$\nu = \nu_0\sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}} = \nu_0\sqrt{\frac{1 - \frac{\frac{a't^*}{c}}{\sqrt{1 + \left(\frac{a't^*}{c}\right)^2}}}{1 + \frac{\frac{a't^*}{c}}{\sqrt{1 + \left(\frac{a't^*}{c}\right)^2}}}} = \nu_0\sqrt{\frac{\sqrt{1 + \left(\frac{a't^*}{c}\right)^2} - \frac{a't^*}{c}}{\sqrt{1 + \left(\frac{a't^*}{c}\right)^2} + \frac{a't^*}{c}}} = \nu_0\left[\sqrt{1 + \left(\frac{a't^*}{c}\right)^2} - \frac{a't^*}{c}\right]$$

But

$$ct^* = \frac{c^2}{a'}\sinh\tau \Rightarrow \frac{a't^*}{c} = \sinh\tau \Rightarrow \nu = \nu_0(\cosh\tau - \sinh\tau) = \nu_0\,\mathrm{e}^{-\tau} = \nu_0\left(1 - \frac{ct}{x_0}\right)$$

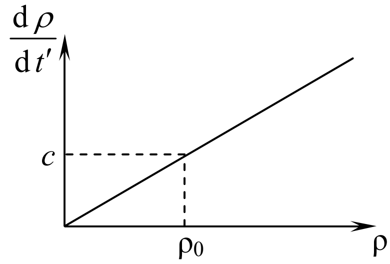
$$t = NT_0 \Rightarrow \nu = \nu_0 \left\{ 1 - \frac{cT_0}{x_0} \left(\left[\frac{x_0}{cT_0} \right] + 1 \right) \right\}$$

l. 0.5p

$$\frac{\mathrm{d}\rho}{\mathrm{d}t'} = \frac{\mathrm{d}\rho}{\mathrm{d}\tau}\frac{\mathrm{d}\tau}{\mathrm{d}t'} = (x_0 - ct)\mathrm{e}^{\tau}\frac{a'}{c} = \frac{a'}{c}\rho$$

Upon reception,

$$\mathrm{e}^{\tau} = \frac{x_0}{x_0 - ct} \Rightarrow \frac{d\rho}{dt'} = (x_0 - ct)\frac{x_0}{x_0 - ct}\frac{a'}{c} = \frac{c^2}{a'}\frac{a'}{c} = c$$



m. 1p

$$\tanh\tau = \frac{ct}{x_0} \Rightarrow \frac{1}{\cosh^2\tau}\mathrm{d}\tau = \frac{c}{x_0}\mathrm{d}t \Rightarrow \mathrm{d}t = \frac{x_0}{c}\left(1 - \tanh^2\tau\right)\mathrm{d}\tau = \frac{x_0}{c}\left(1 - \frac{c^2t^2}{x_0^2}\right)\mathrm{d}\tau$$

$$\frac{\mathrm{d}(\mathrm{d}t)}{\mathrm{d}x_0} = \frac{\mathrm{d}\tau}{c}\frac{\mathrm{d}\left(\frac{x_0^2 - c^2t^2}{x_0}\right)}{\mathrm{d}x_0} = \frac{\mathrm{d}\tau}{c}\frac{x_0^2 + c^2t^2}{x_0^2}$$

$$\varepsilon = \frac{\frac{\mathrm{d}\tau}{c}\frac{x_0^2 + c^2t^2}{x_0^2}\Delta x_0}{\frac{\mathrm{d}\tau}{c}\frac{x_0^2 - c^2t^2}{x_0}} = \frac{x_0^2 + c^2t^2}{x_0^2 - c^2t^2}\frac{\Delta x_0}{x_0}$$

n. 0.5p

$$\varepsilon = \frac{\Delta x_0}{x_0} = \frac{h}{\frac{c^2}{a'}} = \frac{gh}{c^2} \approx \frac{10\,\mathrm{m/s^2} \cdot 360 \cdot 10^3\,\mathrm{km}}{9 \cdot 10^{16}\,\mathrm{m^2/s^2}} = 4 \cdot 10^{-11}$$

$$\Delta t = 4 \cdot 10^{-11} \cdot 365 \cdot 24 \cdot 3600 \approx 1.26 \cdot 10^{-3}\,\mathrm{s}$$