1. Determining the values of the parameters of a real diode by studying its current-voltage characteristic

A theoretical model is a number of mathematical equations which relate the quantities involved in a physical phenomenon. The validity of a model consists in the concordance between the measured data and the predicted values.
The simplest situation which can occur in a model is the case when two quantities $(y_{\text{model}}, x_{\text{model}})$ *are in a linear dependency. It is to be expected that the experimental pairs of measured values* $\left(x^i_{measured}, y^i_{measured}\right)$, $i=\overline{1,N}$ *are subject to imprecision. This is why the plotted pairs of coordinates form a "cloud" around the predicted straight line, called fitting line.*
It is assumed that the best fit of the experimental data with the theoretical model is attained when the sum of the squared distances from these points to the predicted line has the smallest possible value. The slope a *and the slope-intercept* b *for a linear model are given by*

$$a = \frac{N\sum_{i=1}^{N} x_i \cdot y_i - \sum_{i=1}^{N} x_i \cdot \sum_{i=1}^{N} y_i}{N \cdot \sum_{i=1}^{N} x_i^2 - \left(\sum_{i=1}^{N} x_i\right)^2} \qquad (1)$$

$$b = \frac{\sum_{i=1}^{N} x_i^2 \cdot \sum_{i=1}^{N} y_i - \sum_{i=1}^{N} x_i \cdot \sum_{i=1}^{N} x_i \cdot y_i}{N \cdot \sum_{i=1}^{N} x_i^2 - \left(\sum_{i=1}^{N} x_i\right)^2} \qquad (2)$$

Whether there really is a linear dependency between the two quantities is a matter described by the so-called correlation coefficient G.

$$G = \frac{N \cdot \sum_{i=1}^{N} x_i \cdot y_i - \sum_{i=1}^{N} x_i \cdot \sum_{i=1}^{N} y_i}{\sqrt{\left|N \cdot \sum_{i=1}^{N} x_i^2 - \left(\sum_{i=1}^{N} x_i\right)^2\right| \cdot \left|N \cdot \sum_{i=1}^{N} y_i^2 - \left(\sum_{i=1}^{N} y_i\right)^2\right|}} \qquad (3)$$

$G=+1$ *if there truly is a linear dependency with positive slope.* $G=-1$ *if there truly is a linear dependency with negative slope. In general,* $-1 \le G \le 1$ *and the more G gets closer to 0, the less credible the linear dependency is.*
Very few of the physical models imply a linear dependency, but often a suitable rearrangement of a mathematical function can lead to one. The relation between the current I *and the voltage* V *for an ideal diode is given by*

$$I = I_0 \cdot \left(e^{\frac{qV}{\eta \cdot k_B \cdot T}} - 1 \right) \quad (4)$$

where $q = 1.60 \times 10^{-19} C$ *is the elementary electrical charge, and* $k_B = 1.38 \times 10^{-23} J \cdot K^{-1}$ *is the Boltzmann constant. For ordinary experimental conditions,* $T = 300K$ *and* $q/(k_B T) = 38.64 V^{-1}$. *The quantities which determine the non-linear dependency of I with V are the* diode factor η *and the saturation current* I_0. *The curve I versus V looks something like in Figure 1. When subject to large enough positive voltages, the diode's resistance can be considered zero. When subject to large enough negative voltages, the diode's resistance can be considered infinite.*

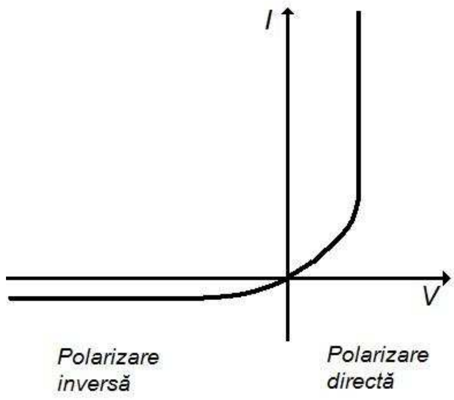

Figure 1. Schematic curve for the I-V characteristic of an ideal diode

Admitting that $0{,}5 \le \eta \le 3$, *for* $V < 0$ *we get* $qV/(\eta \cdot k_B \cdot T) << 0$ *and thus* $e^{\frac{qV}{\eta \cdot k_B \cdot T}} \cong 0$. *So, for reversed polarity, (4) reduces to*

$$I = -I_0 \quad (5)$$

Plotting the I-V characteristic for negative polarity we get a line parallel to the V-axis, which allows us to determine the value of I_0.

For $V > 0$, $qV/(\eta \cdot k_B \cdot T) >> 0$ *and thus* $e^{\frac{qV}{\eta \cdot k_B \cdot T}} >> 1$. *Consequently, (4) becomes*

$$I = I_0 \cdot e^{\frac{qV}{\eta \cdot k_B \cdot T}} \quad (6)$$

or

$$\ln(I) = \ln(I_0) + \frac{qV}{\eta k_B T} \quad (7)$$

As you can see, writing the I-V characteristic as $\ln(I) = f(V)$, *we get a linear dependency. The slope allows us to determine the value of the diode factor, while the slope-intercept allows us to determine the value of the saturation current.*
One can consider that a „real" diode consists of an ideal diode connected in parallel with a shunt with resistance R, and the whole connected in series with a resistor r. *Tipical values for the characteristic resistances of r and R are* $0.01\Omega < r < 10\Omega$, *and* $75k\Omega < R < 1M\Omega$ *respectively.*

The I-V characteristic of a real diode is given by

$$I = I_0 \cdot \left(e^{\frac{q(V - r \cdot I)}{\eta \cdot k_B \cdot T}} - 1 \right) + \frac{V - r \cdot I}{R} \tag{8}$$

Experimental problem 1 asks you to determine the values of the parameters R, r, I_0, η of a diode, by using the data in the following two tables.

Table 1. Pairs of experimental data current-voltage for a real diode with reversed polarity

No.	U(V)	I(μA)	Nr.	U(V)	I(μA)	Nr.	U(V)	I(μA)	Nr.	U(V)	I(μA)
1	0	0	9	-0.5	-11	16	-1.2	-12	24	-2	-14
2	-0.05	-6	10	-0.6	-11	17	-1.3	-13	25	-2.05	-14
3	-0.1	-9	11	-0.7	-11	18	-1.4	-13	26	-2.1	-14
4	-0.15	-10	12	-0.8	-12	19	-1.5	-13	27	-2.2	-14
5	-0.2	-10	13	-0.9	-12	20	-1.6	-13	28	-2.3	-15
6	-0.25	-10	14	-1	-12	21	-1.7	-13	29	-2.4	-15
7	-0.3	-11	15	-1.1	-12	22	-1.8	-14	30	-2.5	-15
8	-0.4	-11				23	-1.9	-14			

In order to determine the values of the parameters of the diode for reversed polarity follow de below procedure:

1. In the corresponding box on the answer sheet write down the expression of (6) for large values of the negative voltage $V << 0$.
2. Using the experimental data, plot on the provided milimetric paper the I-V curve for reversed polarity.
3. In the corresponding box on the answer sheet write down the linear form of the I-V characteristic for reversed polarity and indicate the expressions of the slope a and of the slope-intercept b.
4. Neglect from the experimental data the entries corresponding to small values of the negative voltage, and fill in Table 3 on the answer sheet.
5. Determine the parameters of the fitting line and fill in the corresponding boxes on the answer sheet.
6. Determine the correlation coefficient and write it down in the corresponding box on the answer sheet.
7. Determine the values of the parameters R, I_0 and write them down in the corresponding boxes on the answer sheet.

Table 2. Pairs of experimental data current-voltage for a real diode with forward polarity

No.	Voltage (V)	Current (A)	No.	Voltage (V)	Current (A)
1	0.345	0.007	11	0.491	0.082
2	0.383	0.015	12	0.498	0.090
3	0.406	0.022	13	0.504	0.097
4	0.423	0.030	14	0.511	0.105
5	0.437	0.037	15	0.516	0.112
6	0.449	0.045	16	0.522	0.120
7	0.459	0.052	17	0.527	0.127
8	0.468	0.060	18	0.533	0.135
9	0.477	0.067	19	0.538	0.142
10	0.484	0.075	20	0.543	0.150

In order to determine the values of the parameters of the diode for forward polarity follow de below procedure:

8. In the corresponding box on the answer sheet write down the expression of (6) for large values of the positive voltage, $V >> 0$.
9. Using the experimental data, plot on the provided milimetric paper the graph $\ln(I) = f(V)$ for forward polarity.
10. Fill in Table 4 on the answer sheet only with the experimental data corresponding to moderate values of the positive voltage.
11. Determine the parameters of the fitting line and fill in the corresponding boxes on the answer sheet.
12. Determine the correlation coefficient and write it down in the corresponding box on the answer sheet.
13. Draw on the graph plotted at point 9 the resulting fitting line.
14. Determine the values of the parameters η, I_0 and write them down in the corresponding boxes on the answer sheet.
15. The measured data for large positive voltages lay below the fitting line. Try to provide an explanation for this fact
16. Determine the value of the resistor r and write it down in the corresponding box on the answer sheet.

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Answer Sheet

1. The expression of (6) for large values of the negative voltage:

2. The $I-V$ graph: drawn on the provided milimetric paper.

3. The linear form of the $I-V$ curve for reversed polarity:

The expression for the slope a:

The expression for the slope-intercept b:

4. Table 3 – Processed data for reversed polarity

No.	U_i	I_i	U_i^2	I_i^2	$U_i \cdot I_i$
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
Σ					

5. The parameters of the fitting line

The slope of the fitting line:

The slope-intercept of the fitting line:

6. The correlation coefficient :

7. The value of the parameter I_0 :

The value of the parameter R :

8. The form of (6) for large values of the positive voltage :

9. The $\ln(I) = f(V)$ graph for forward polarity: drawn on the provided milimetric paper.

10. Table 4 - Processed data for forward polarity

No.	$U(V)$	$I(A)$	$\ln(I)$	U^2	$(\ln(I))^2$	$U \cdot \ln(I)$
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
Σ						

11. The parameters of the fitting line

The slope of the fitting line

The slope-intercept of the fitting line

12. The correlation coefficient

13. The drawing of the resulting line on the graph plotted at point 9.

14. The expression of the parameter η :

The value of the parameter η :

The expression of the parameter I_0 :

The value of the parameter I_0 :

15. The explanation you provide for the fact that the measured data for large positive voltages lay below the fitting line.

16. The value of the resistor r

2. Study of the Moiré patterns

When two transparent layers containing correlated opaque patterns are superposed, a Moiré pattern appears. The simplest Moiré pattern can be observed when a transparency comprising a grid (periodically repeating opaque parallel lines) is superposed onto a paper sheet comprising also a grid as in the Figure 1.

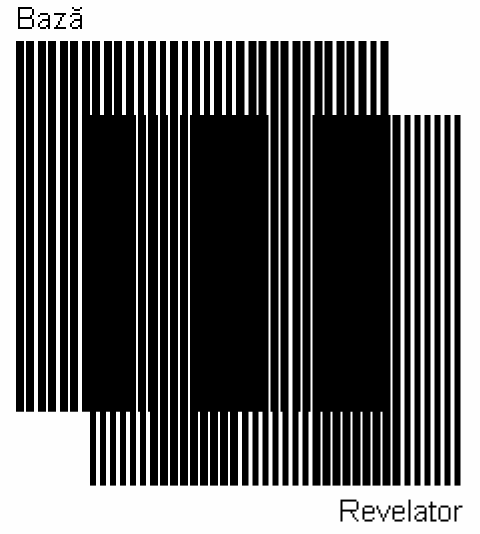

Figure 2

The period of a grid is the space between the axes of two adjacent parallel lines; in a Moiré experiment the difference between the periods of the two grids must be very small.
*In what follows, the base sheet will be named **"base"** and the transparency will be named **"revelator".** Let's denote the period of the "base" by p_1, and the period of the "revelator" by p_2. The image appearing by superposition of the grids outlines periodically repeating dark parallel bands, called Moiré lines. The spacing between the Moiré lines is much larger than the periods of the two grids.*
The light areas of the superposed image correspond to the zones where the lines of the two grids overlap. The dark areas of the superposition image, forming the Moiré lines, correspond to the places where a dark line of one grid overlaps with a white space between the lines of the other grid. This can be seen in the Figure 2.

The dark and light Moiré lines are periodically interchanging. In the case shown $p_2 < p_1$, so if we start counting layer lines from the point where they overlap, the base layer lines, having a longer period, advance faster than the revelator layer lines, which have a shorter period. By the distance p_M, the base layer lines are ahead of the revelator layer lines by a full period, and the lines of the two layers overlap again.

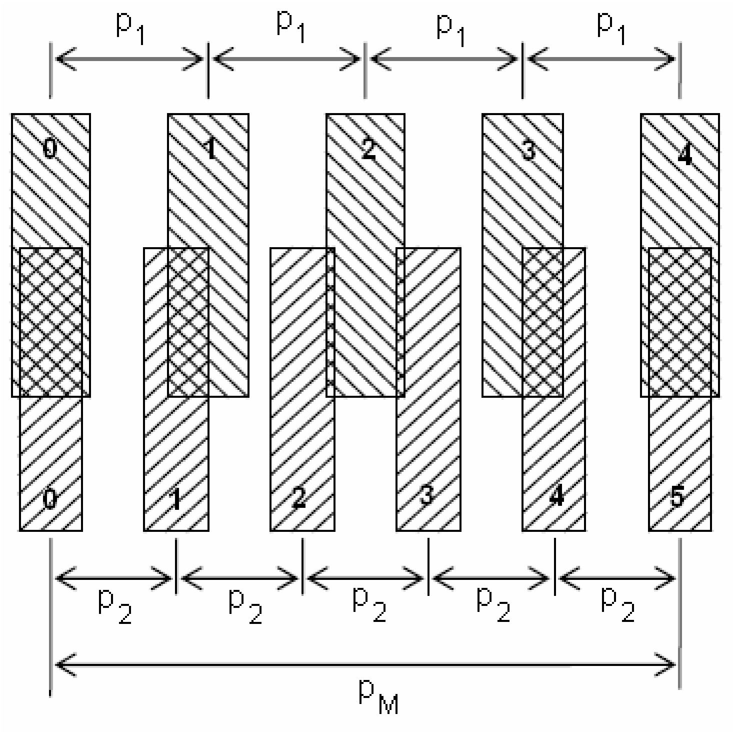

Figure 3

The number p_M / p_1 of base layer lines contained in the distance p_M is equal to the number p_M / p_2 of revealing layer lines contained in the distance p_M minus one

$$\frac{p_M}{p_1} = \frac{p_M}{p_2} - 1$$
$$\frac{1}{p_M} = \frac{1}{p_2} - \frac{1}{p_1} \qquad (9)$$

that is

$$p_M = \frac{p_1 \cdot p_2}{p_1 - p_2} \qquad (10)$$

The superposition of two layers comprising grids of parallel lines forms an optical image with parallel Moiré lines with a augmented period. The closer the period of the two grids (i.e. the smaller the difference $p_1 - p_2$) the bigger the period of Moiré pattern.
If the situation is reversed and the revelator grid period is longer than the base layer period, the space between the Moiré lines is equal to the absolute value of formula (2).

Moiré patterns are useful in representing fluid flow and potential fields. Problems in optics, wave motion, stress analysis, crystallography, mathematics, and the psychology of perception may also be solved.

Experimental set-up

For experimental problem 2 you receive a kit containing:

1. An expanded Styrofoam board serving as experimental table, allowing the thrusting (pricking) of pins.
2. An envelope containing :
   - **a.** A transparency marked "Rigla, raportor, grila 1". On the transparency it is printed:
     - i. a protractor;
     - ii. a ruler with major ticks divided in ten minor ticks; this minor ticks represent the arbitrary unit (a.u.) of the experiment;
     - iii. a marked grid, whose period is 0.76 a.u.
   - **b.** A transparency marked "Rigla, raportor, grila 2".
   - **c.** A paper sheet marked "Rigla, raportor, grila 2".
   - **d.** A paper sheet marked "Rigla, raportor, grila 3"
   - **e.** Four clear transparencies. (Attention: you are allowed to write or to draw **only** on these transparencies).
3. A small plastic bag containing:
   - **a.** A few pins.
   - **b.** Two different colors markers.

(When you use the transparencies detach them from the protecting paper sheet.)

Experimental Questions

I. Parallel grid

A. Using the transparency marked "Rigla, raportor, grila 1" and the paper sheet marked "Rigla, raportor, grila 2", a clear transparency, pins and markers, determine in a. u. the period of the grid printed on paper. Write in the appropriate box on the Answer Sheet the value in a. u. of the period of the Moiré pattern. Write in the appropriate box on the Answer Sheet the value in a. u. of the period of the grid printed on paper.

B. Using the transparency marked "Rigla, raportor, grila 1 and the paper sheet marked "Rigla, raportor, grila 3", a clear transparency, pins and markers, determine the period of the grid printed on paper. Write in the appropriate box on the Answer Sheet the value in a. u. of the period of the Moiré pattern. Write in the appropriate box on the Answer Sheet the value in a. u. of the period of the grid printed on paper.

C. Test for the possibility of appearance of a Moiré pattern using the transparency marked "Rigla, raportor, grila 2" and the paper sheet marked "Rigla, raportor, grila 2". In the appropriated box on the Answer Sheet write a comment about the observed image.

D. Examining the grids and using the obtained results, write the values of the periods of grids in the appropriate boxes on the Answer Sheet.

II Rotated grids

E. Let us consider two grids with the same period p, but the two grids are rotated by an angle α with respect to each other. Find the expression of the period of the appearing Moiré pattern in terms of p and α.

F. Using the transparency marked "Rigla, raportor, grila 2" and the paper sheet marked "Rigla, raportor, grila 2", a clear transparency, pins and markers, measure the period of the Moiré pattern when the angle between the grids increase from 0° to 15°. At least 15 measurements are required. Fill in the Table 1 on the Answer Sheet.

G. Fill in the Table 2 with the theoretical and measured values of the Moiré patterns periods and with the ratio of these two periods. At least 10 set of data are required.

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Answer Sheet

I. Parallel grid

A. The value in a. u. of the period of the Moiré pattern

The value in a. u. of the period of the grid printed on the "Rigla, raportor, grila 2" sheet

B. The value in a. u. of the period of the Moiré pattern

The value in a. u. of the period of the grid printed on the "Rigla, raportor, grila 3" sheet

C. Comment on the observed image

D. The values of the periods of the grids

"Rigla, raportor, grila 1" a.u.	"Rigla, raportor, grila 2" a.u.	"Rigla, raportor, grila 3" a.u.
0,76 a.u.		

E. The expression of the period of the appearing Moiré pattern in terms of p and α

F. Table 1

No.	p_M	α

G. Table 2

No.	α	$p_{M,\text{theoretical}}$	$p_{M,\text{experimental}}$	$\frac{p_{M,\text{theoretical}}}{p_{M,\text{experimental}}}$